Basic Radiance Theory

The laws of geometric optics and radiance transforms

Adobe

Ray Transforms

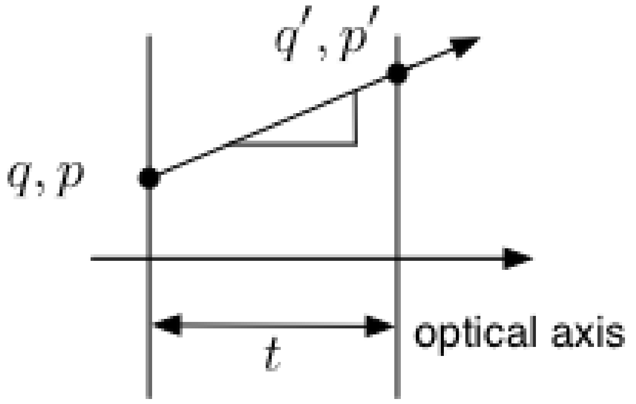
The main laws of geometric optics

Phase Space

- This is the (q, p) space of rays. It is a 4D vector space with zero vector the optical axis.
- Each ray is a 4D point (a vector) in that space.
- Any optical device, like a microscope or a telescope, is a matrix that transforms an incoming ray into an outgoing ray.
- This matrix can be computed as a product of the optical elements that make up the device.

Summary: Two Primary Optical Transforms

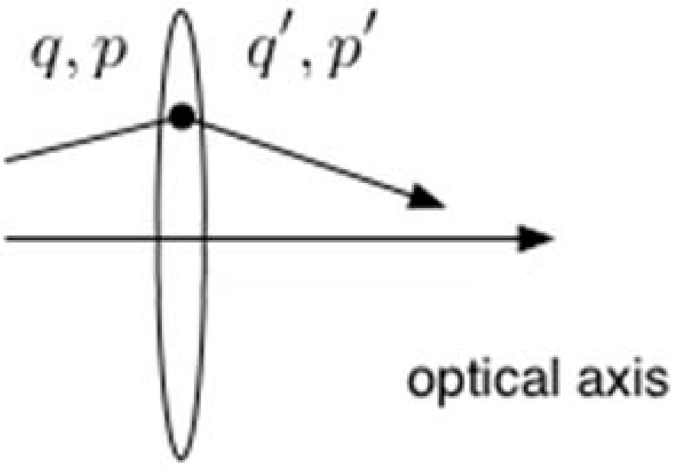
Transport



$$\begin{bmatrix} q' \\ p' \end{bmatrix} = T \begin{bmatrix} q \\ p \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

Lens

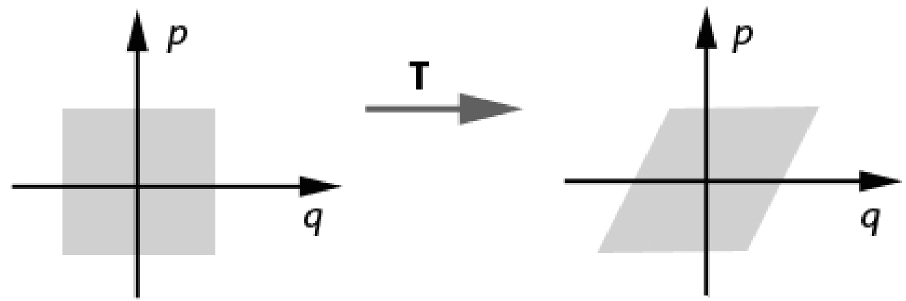


$$\begin{bmatrix} q' \\ p' \end{bmatrix} = L \begin{bmatrix} q \\ p \end{bmatrix}$$

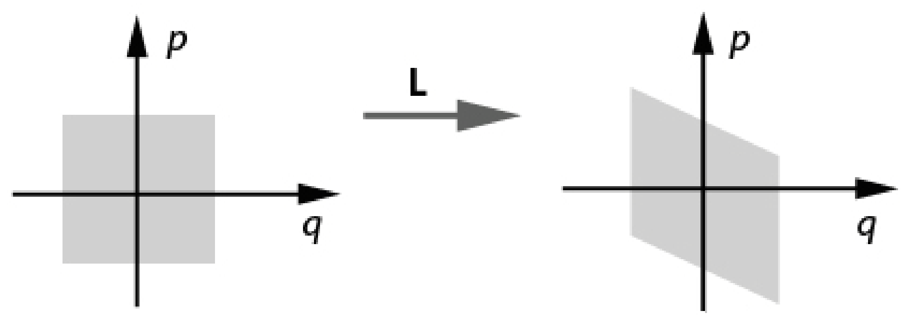
$$L = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

Transformations in Phase Space

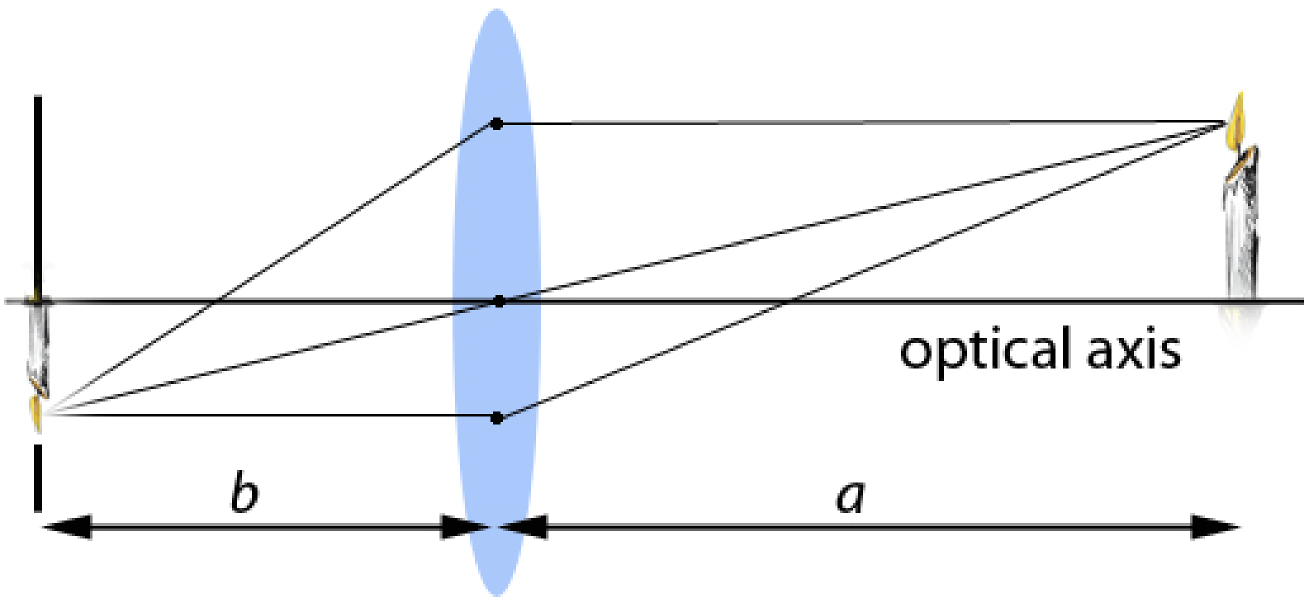
Space transport



Lens refraction



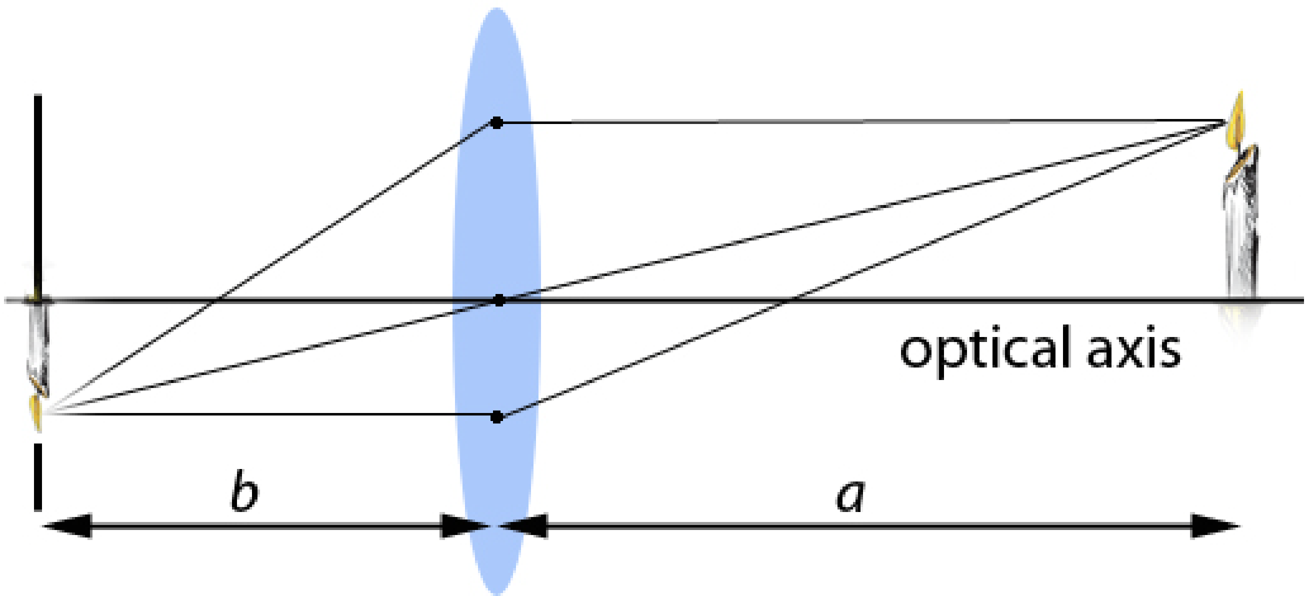
Example: Traditional Camera



$$A = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

Adobe

Traditional Camera



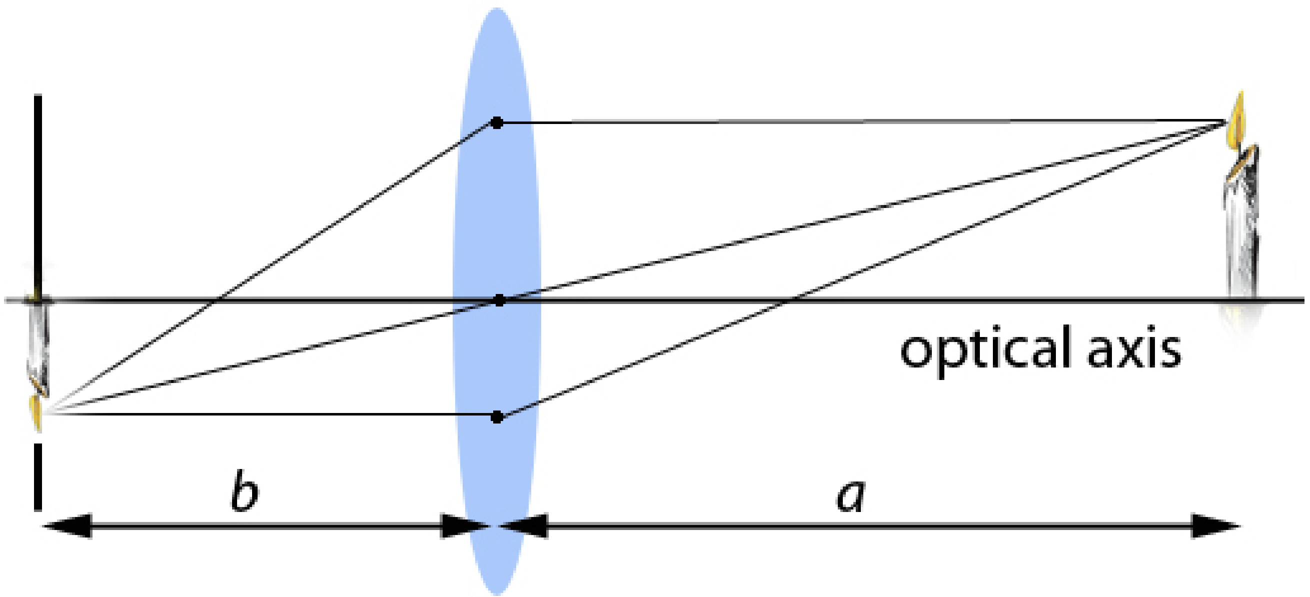
How do we focus?

Make top-right element to be zero

$$A = \begin{bmatrix} 1-\frac{b}{f} & ab\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{f}\right) \\ -\frac{1}{f} & 1-\frac{a}{f} \end{bmatrix}$$

$$= \begin{bmatrix} 1-\frac{b}{f} & 0 \\ -\frac{1}{f} & 1-\frac{a}{f} \end{bmatrix}$$

Traditional Camera



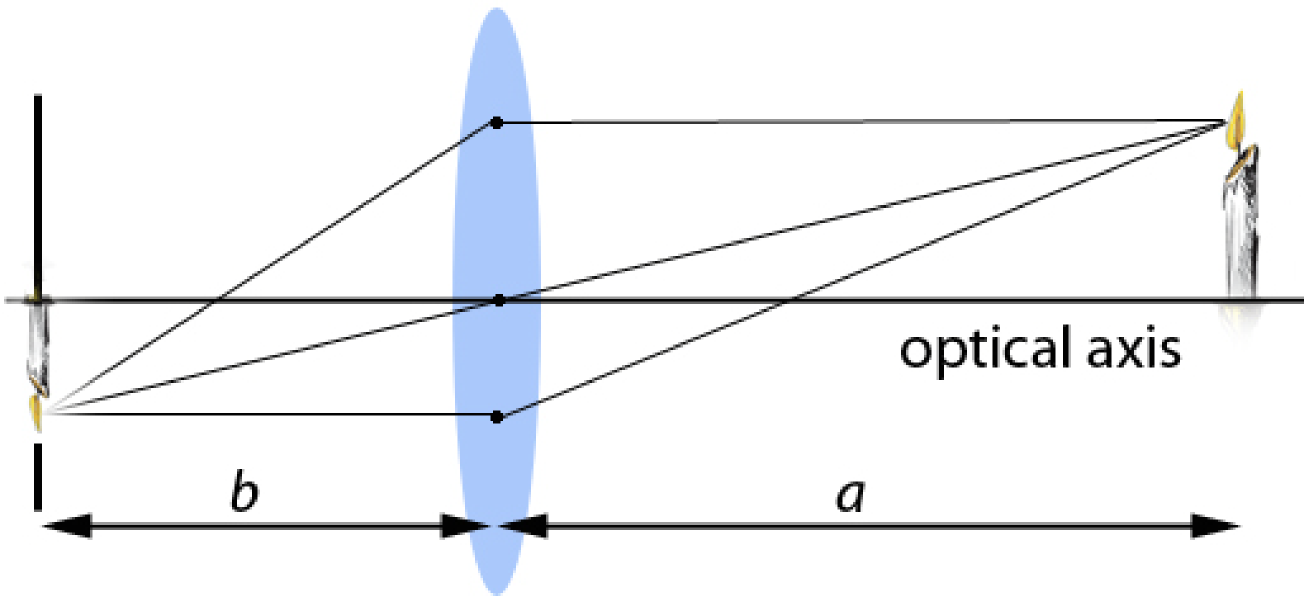
We enforce this condition by:

$$\frac{1}{a} + \frac{1}{b} - \frac{1}{f} = 0$$

$$A = \begin{bmatrix} 1 - \frac{b}{f} & ab\left(\frac{1}{a} + \frac{1}{b} - \frac{1}{f}\right) \\ -\frac{1}{f} & 1 - \frac{a}{f} \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \frac{b}{f} & 0 \\ -\frac{1}{f} & 1 - \frac{a}{f} \end{bmatrix}$$

Traditional Camera



We have *derived* the lens equation: $$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

$$A = \begin{bmatrix} -\frac{b}{a} & 0 \\ -\frac{1}{f} & -\frac{a}{b} \end{bmatrix}$$

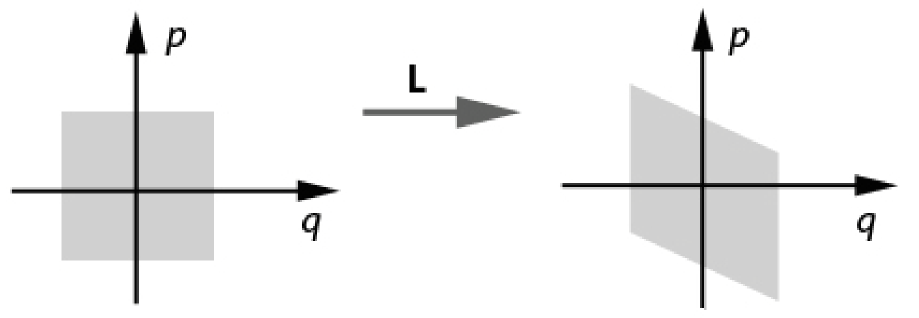
Radiance

Definition and main mathematical properties

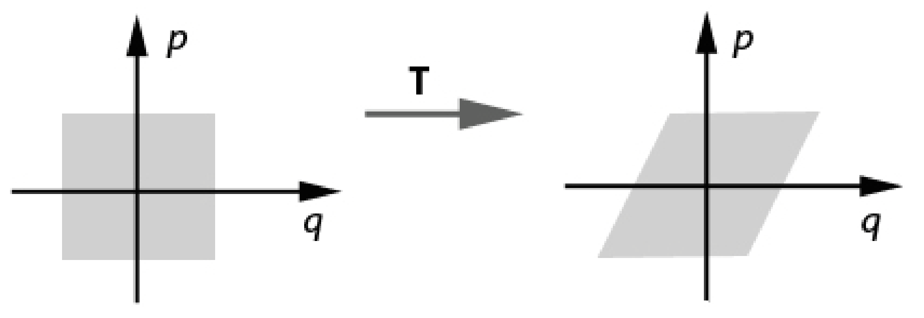
Conservation of Volume

- For the 2 transforms, the 4D box changes shape, but volume remains the same (shear)
- Volume is conserved for any optical transform!

$$L = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$



$$T = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

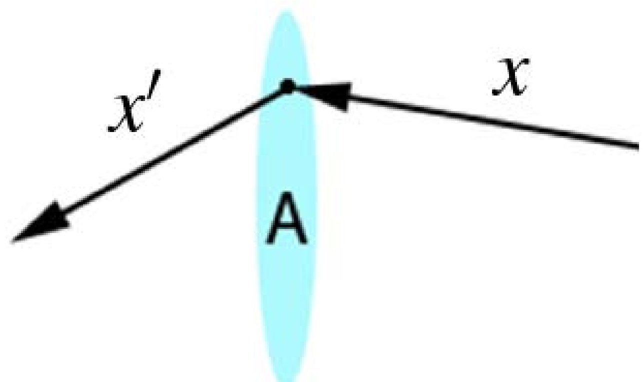


Conservation of Radiance

- Radiance is energy density in 4D ray-space
- Energy is conserved; volume is conserved
- Radiance = (energy) / (volume)
- ***Radiance is also conserved!***

Radiance Transforms

$$x = \begin{bmatrix} q \\ p \end{bmatrix}$$



- Radiance before optical transform $r(x)$
- Radiance after optical transform $r'(x)$

Radiance Transforms

$$x' = Ax$$

Due to radiance conservation,

$$r'(x') = r(x)$$

$$r'(x') = r(A^{-1}x')$$

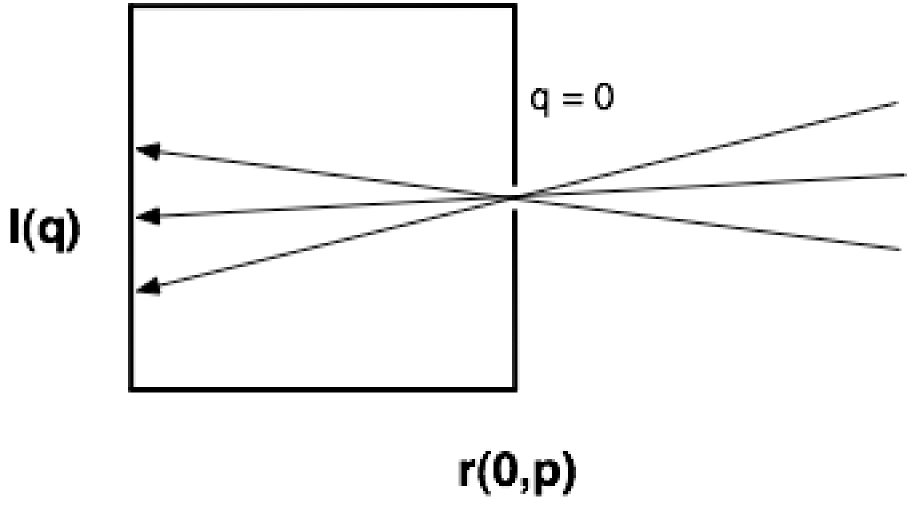
Since x' is arbitrary, we can replace it by x

$$r'(x) = r(A^{-1}x)$$

Capturing Radiance with Cameras

Pinhole Camera

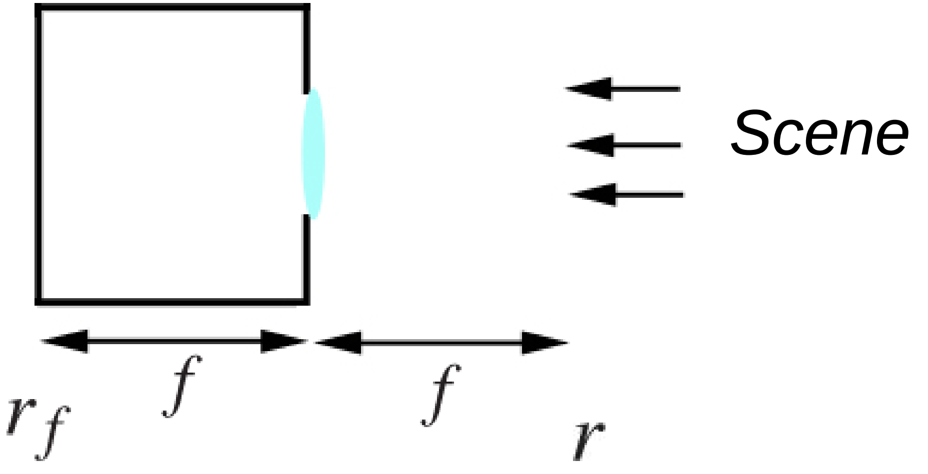
Rays from different directions spread apart inside camera and are captured at different positions on the sensor



- Switches direction and position
- Captures angular distribution of radiance @pinhole

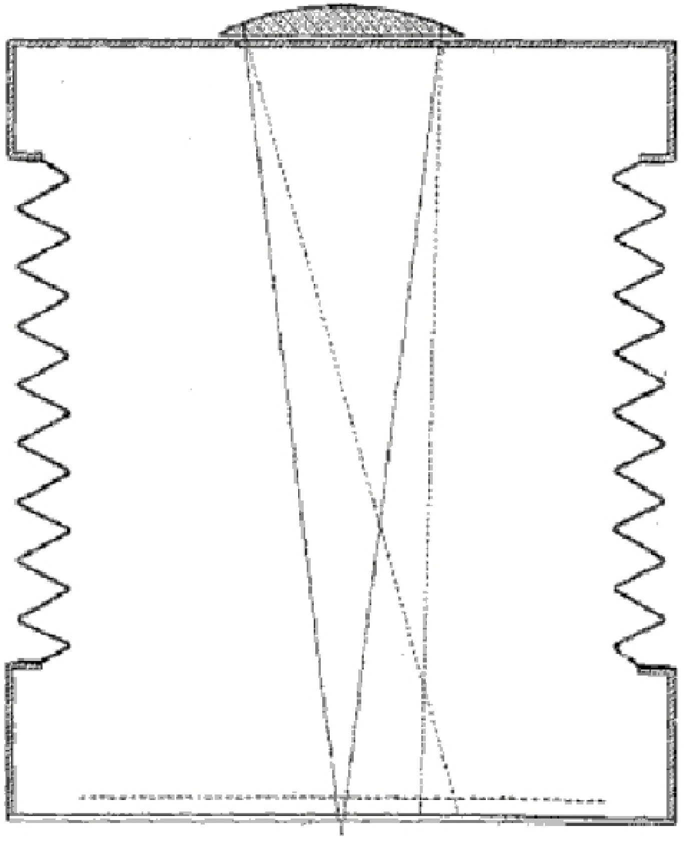
"2F" Camera

- This is the lens generalization of the pinhole camera
- Three optical elements: space, lens, space

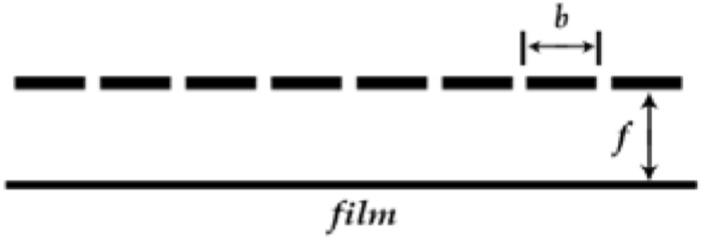


$$A = T_f L_f T_f$$

Ives' Camera (based on the pinhole camera)



At the image plane:

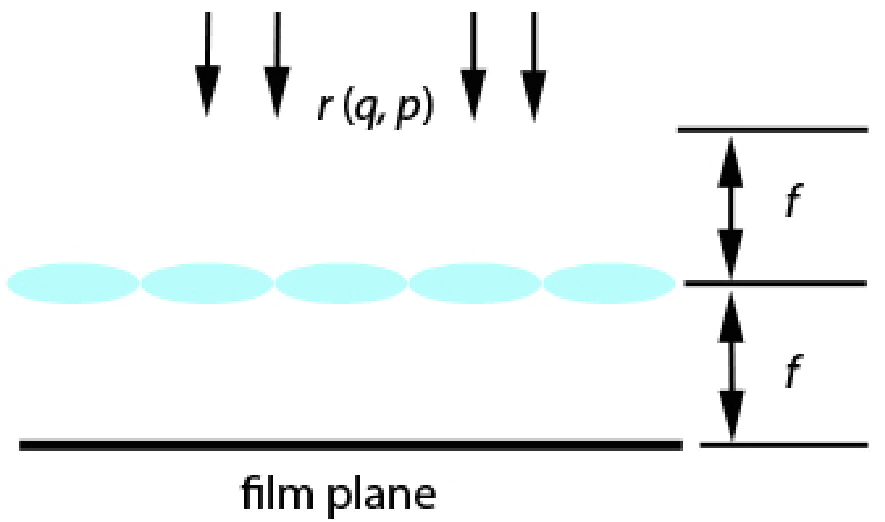


Multiplexing:

Each pinhole image captures angular distribution of radiance. All images together describe the complete 4D radiance.

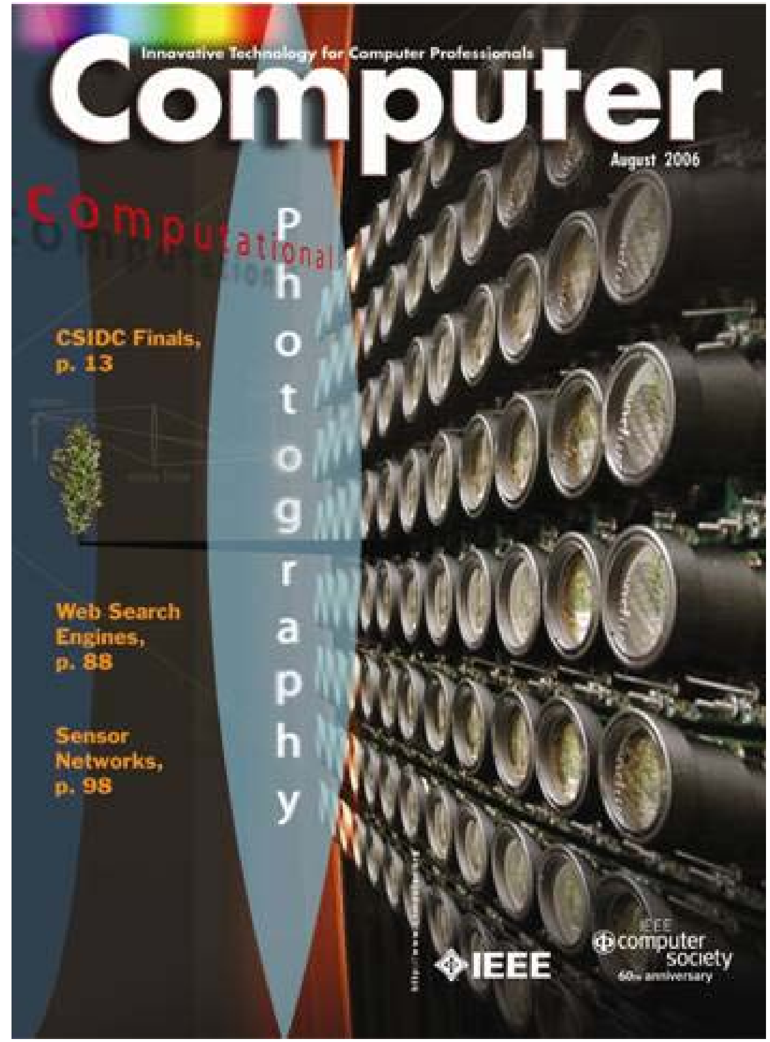
Lippmann's Camera (based on 2F)

- Multiplexing
- Lenses instead of pinholes
- 2F cameras replaces the pinhole camera



Camera Arrays

- The most popular lightfield camera is simply an array of conventional cameras, like the Stanford array.
- Alternatively, an array of lenses/prisms with a common sensor, like the Adobe array.



Adobe

Adobe Array of Lenses and Prisms

Adobe

Adobe Array of Lenses and Prisms

Arrays of Lenses and Prisms

Shifting cameras from the optical axis means: We need to extend the vector space treatment to affine space treatment.

Prism transform

$$\begin{pmatrix} q' \\ p' \end{pmatrix} = \begin{pmatrix} q \\ p \end{pmatrix} + \begin{pmatrix} 0 \\ \alpha \end{pmatrix}$$

Shifted lens

$$\begin{pmatrix} q' \\ p' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} q - s \\ p \end{pmatrix} + \begin{pmatrix} s \\ 0 \end{pmatrix}$$

Lens + prism

$$\begin{pmatrix} q' \\ p' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} q \\ p \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{s}{f} \end{pmatrix}$$

Refocusing with the Adobe camera

Refocusing with the Adobe camera