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Effect of three-dimensional moiré in integral photography

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An attempt is made to explain certain properties of an integral image, and in particular, the character of the alternation in the associated integral images by using the theory of the three-dimensional moiré effect.

The method of integral photography is based on the use of the so-called integral plate, usually consisting of a raster with spherical lens elements and a photoplate.[1] When the picture is taken, each of these elements forms its microscopic image on the layer. After the picture is taken, the plate is treated photographically, then illuminated with scattered light on the side of the emulsion layer, and the observer can view the reconstructed two-dimensional integral image of the object with the unaided eye.[1]

Analysis of certain phenomena taking place during the reconstruction of the integral image has made it necessary to postulate that they may be treated as the result of a three-dimensional moiré effect,[2] which differs from the commonly known moiré effect in the fact that the moiré patterns are localized, not at the boundary of two structures, but at a certain distance from them. This hypothesis accounts for a number of properties of an integral image, manifested during the assembly and adjustment of an integral plate. In this connection, it was desirable to use a recently evolved[3] theory of the three-dimensional moiré effect to account for these properties.

The raster of an integral plate usually consists of a periodic structure with a definite alternation step of the elements. Since all the elements form their microimages on the photosensitive layer, after the treatment, a series of microimages, which constitutes a periodic structure with a spacing differing insignificantly from the raster spacing will be formed on this layer. Usually, the raster plane of an integral plate is parallel to the plane of the photosensitive layer and separated from it by an air gap. Thus, when an integral image is reconstructed, all the components necessary for the moiré effect to occur are available: scattered light and two periodic structures having an insignificantly different spacing, located in two planes separated by an air gap. In addition, in real rasters used in obtaining integral photographs, there are no longitudinal partitions separating the regions of formation of microimages by each element, so that the rays can penetrate unimpeded from one region to the other.

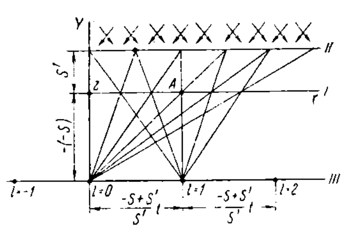


FIG. 1. Diagram of formation of the principal and associated images during the formation of the three-dimensional moiré effect: I-raster, II-treated photosensitive layer, III-plane of integral images.

We will assume for simplicity that photographic treatment with reversal was applied to an integral plate consisting of a system of small pin holes giving a diffraction image of the object. If the object is a self-luminous point, after photographic treatment with reversal, the photosensitive layer will have recorded microimages of the object which are transparent points on a black background. We will reconstruct the integral image by illuminating the raster-photosensitive layer system with scattered light from the side of the photosensitive layer (Fig. 1). Assuming that in our case the moiré pattern formed may be considered the image of the object point, we will determine the localization of this pattern relative to raster I. We analyze the problem by using analytical geometry. A similar method was used by Feokaris *et al.*[3] in a study of the three-dimensional moiré effect for two amplitude gratings with parallel rulings of similar spacing; their planes were assumed to be inclined toward one another or parallel.

We will take a raster-photosensitive layer system with a set of rows of transparent points on the photosensitive layer and on the raster plane. We will assume that these rows are directed along the X axis (Fig. 1), and the distance between the raster and the photosensi-

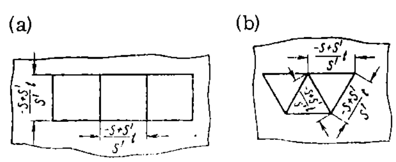


FIG. 2. Location of luminous points—three-dimensional moiré patterns in the integral image plane: a-arrangement of raster lenses in rows, b-honeycomb arrangement of raster lenses.

Location of Luminous Points in the Image Plane

l	0	1	2	3	4	5	6
X	0	$1\frac{-S+S'}{S'}t$	$2\frac{-S+S'}{S'}t$	$3\frac{-S+S'}{S'}t$	$4\frac{-S+S'}{S'}t$	$5\frac{-S+S'}{S'}t$	$6\frac{-S+S'}{S'}t$
Y	S	S	S	S	S	S	S

tive layer corresponds to the formation of the microimages by the system of small holes. Let the raster spacing be t, and the spacing of rows of transparent points on the photosensitive layer be t', t' being related to the relation resulting from an inspection of Fig. 1 as follows:

$$t'=\frac{-S+S'}{-S}t, \tag{1}$$

$$\Delta t=t'-t=\frac{S'}{-S}t. \tag{2}$$

Since $-S \gg S'$, t' insignificantly exceeds t, so that distinct moiré patterns can be obtained. As we know, the formation of a moiré pattern takes place where the accumulating difference between the length of the reference row and that of the row being compared reaches the magnitude of one spacing.

Let A and B be the points of rows differing by the 1-th pattern number. Then their coordinates may be written in the form

$$A(mt,0),\ B[t(m-l)(1+\Delta t),\ S'],$$

where $m=0,\ 1,\ 2,\ 3,\ \cdots$ is the number of the point of row I. We will assume that $m=1=0$ when $X=0$. Then the equation of the straight line passing through points A and B may be written in the well-known form

$$\begin{vmatrix} X_B-X_A & Y_B-Y_A \\ X-X_A & Y-Y_A \end{vmatrix}=0.$$

Hence we obtain

$$A(m)X-B(m)Y+C(m)=0, \tag{3}$$

where

$$\left.\begin{aligned} A(m)&=S', \\ B(m)&=t[\Delta t(m-l)-l], \\ C(m)&=S'mt. \end{aligned}\right\} \tag{4}$$

Assuming that the intersection of the rays forming the moiré pattern is complex in character, their localization will be found by determining the parametric equations surrounding these intersection points for all l to which all the rays emerging from the raster-photogram unit are tangent. For this purpose, we differentiate expression (3) with respect to m:

$$A'(m)X-B'(m)Y+C'(m)=0. \tag{5}$$

Then on the basis of Eqs. (4) and (5), we express X and Y in terms of m:

$$\left.\begin{aligned} X&=\frac{B(m)C'(m)-B'(m)C(m)}{A(m)B'(m)-A'(m)B(m)}, \\ Y&=\frac{A(m)C'(m)-A'(m)C(m)}{A(m)B'(m)-A'(m)B(m)}. \end{aligned}\right\} \tag{6}$$

Considering the values of $A(m)$, $B(m)$, $C(m)$ from (4) and differentiating them with respect to m, we obtain the general parametric equations of the envelopes, which in our case degenerate into a system of points:

$$\left.\begin{aligned} X&=t\cdot\frac{l(1+\Delta t)}{\Delta t}, \\ Y&=\frac{-S'}{\Delta t} \end{aligned}\right\} \tag{7}$$

or

$$X=\frac{-S+S'}{S'}tl, \tag{8}$$

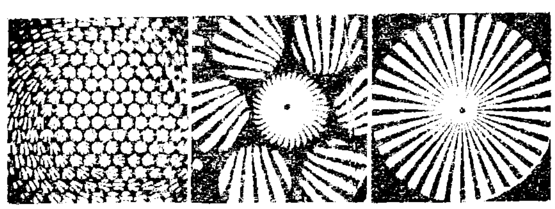
$$Y=-(-S). \tag{9}$$

The minus sign in expression (9) means that the points are located in negative Y quadrants.

Thus, in our case, the three-dimensional moiré patterns degenerate into a system of luminous points, one of which coincides with the location of the object point. This point will represent a zero-order moiré pattern, and the remaining points will represent patterns of higher orders.

In other words, the analysis of the reconstruction of an integral image, performed by using the three-dimensional moiré theory, confirmed the conclusion of a multiplicity of convergence points of beams, resulting from projective geometry and from the theory of raster systems.[4] On this basis it may be assumed that our hypothesis that the moiré patterns formed are a collection of images of the object point corresponds to reality. We will find the location of these points in the XY plane (see Fig. 2 and table). Because of the symmetry of the entire beam and raster-photosensitive layer unit about the Y axis, it is obvious that the location of the points in the XZ plane will be of the same form, i.e., the general distribution of the system of points—integral images of the object point in the XZ plane will have the appearance shown in Fig. 2.

We will further assume that our object consists of two luminous points separated in depth (along the Y axis) in space in front of the raster. The reconstruction process will give rise to two systems of luminous points, two of which coincide with the location of object points which have been there, and the remaining ones are located in two planes parallel to each other and to the raster plane, the alternation on these planes being strictly similar to the regular structure of the raster. The



(a) (b) (c)

FIG. 3. Principal and associated integral images during rotation of the raster plane relative to the plane of the treated photosensitive layer (without inclinations); a and b-intermediate positions, c-adjusted position (the size of the integral image is maximum).

images of all the points of a three-dimensional object will be formed in analogous fashion, i.e., we will find that the process of reconstruction of the image of a three-dimensional object will lead to the appearance of a principal integral image identical to the object in its location in space and dimensions, and to a series of associated images theoretically no different from the principal image. The alternation of these images will be strictly similar to the structure of the raster, and their location will be determined by formulas (8) and (9).

In the conclusion that the process of formation of associated images may be explained by considering the three-dimensional moirê phenomenon is confirmed not only by the theoretical considerations given above, but also by an extensive experimental material. Part of this material has been described previously.[5,6]

The clearest confirmation of the conclusion may apparently be considered to be the phenomena taking place during the adjustment of the position of the raster relative to the treated photosensitive layer (in the case of a dismountable integral plate). These phenomena strictly correspond to the existing descriptions of the process of formation of three-dimensional moirê patterns and are very similar to the phenomena involved in the formation of a plane moirê pattern.[4,7,8] Thus, for example, when the structure of a treated photosensitive layer is superimposed on a raster, there is observed a series of moirê patterns consisting of a large number of reduced integral images (Fig. 3). As follows from the moirê theory, the reduction turns out to be directly proportional to the angle of rotation of the plane of the photosensitive layer relative to the raster plane, the parallelism of the planes and necessary air gap being preserved. When the raster turns relative to the photosensitive layer in the direction corresponding to the relative position of these planes when the picture is taken, an enlargement of the small integral images takes place. In the position corresponding to the position of the raster and photosensitive layer when the picture is taken, the integral images have their largest size, i.e., there is no rotation of the planes of the raster and photosensitive layer relative to one another.

CONCLUSION

Application of the theory of the spatial moirê effect in integral photography makes it possible to substantiate the process of formation of integral images and explain a series of phenomena taking place during adjustment of the raster relative to the photosensitive layer and during the formation of associated images. This theory may also be successfully used in other photographing raster systems.

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Programmed shaping of an axisymmetric nonspherical optical surface with a small-size tool

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For a given kinematics of motion of a flat tool of small size, the paper presents a method of exact calculation of the law of change for the parameters which program the shaping; the load of the tool on the part and rate of their relative displacement.

It is shown that the equidistant shaping of an optical surface by a small tool may be achieved by means of a programmed change of the dynamic treatment parameter, i.e., the load of the grinding tool on the part.[1] Discussed below is the question of programmed shaping with the same tool of nonspherical optical surfaces of small curvature showing deviations from a plane or a sphere. For a given kinematics of motion, the paper describes a method of calculating the law of change for the parameter programming the shaping, i.e., the load of the tool on the part as functions of the relative coordinates of the plane or sphere.

It is well known[2,3] that abrasive shaping of optical surfaces may be controlled by using the experimentally confirmed relationship between the rate of wear of the surface element and the mechanical work expended in this process, the magnitude of this work being chiefly determined by the following parameters: pressure between the rubbing surfaces, their relative velocity, and the time of the treatment. The shaping process may be controlled by means of a programmed change of one of these parameters, while the others are stabilized.

Essentially, this method of control is a method of programmed distribution of mechanical work over the zones of the workpiece for the purpose of obtaining the required nonspherical surface of revolution. The treatment should be carried out by using a very small tool which, however, has finite dimensions of the contact